

Magnetars and the Chiral Plasma Instabilities

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We propose a possible new mechanism for a strong and stable magnetic field of compact stars due to an instability in the presence of a chirality imbalance of electrons—the chiral plasma instability. A large chirality imbalance of electrons inevitably occurs associated with the parity-violating weak process during core collapse of supernovae. We estimate the maximal magnetic field due to this instability to be of order 10^{18} G at the core. This mechanism naturally generates a large magnetic helicity from the chiral asymmetry, which ensures the stability of the large magnetic field.

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Introduction.—The origin of compact stars with the most powerful magnetic field ($\sim 10^{15}$ G on the surface) in the Universe, called magnetars [1], is a mystery in astrophysics. Examples of the possible mechanisms include the fossil field or dynamo hypothesis among others [2, 3]. However, these mechanisms have an important problem that a strong magnetic field produced cannot be sustained for a long time scale [2, 3]. Indeed, a purely poloidal magnetic field [depicted in Fig. 1(a)] typically considered is known to be unstable [4], as numerically confirmed by magnetohydrodynamics [5]. For other issues in these mechanisms, see, e.g., Ref. [3].

It is suggested that *if* nonzero magnetic helicity

$$\mathcal{H} = \int d\mathbf{x} \mathbf{A} \cdot \mathbf{B} \quad (1)$$

(where \mathbf{B} and \mathbf{A} are the magnetic field and vector potential) is produced at the initial configuration for some reason, it can make the magnetic field stable [3]. This is because \mathcal{H} is proportional to the Gauss linking number of the magnetic flux tubes and serves as an approximate conserved quantity [6].¹ For example, linking of poloidal and toroidal magnetic fields [depicted in Fig. 1(c)] has a nonzero magnetic helicity and can exist stably [7]. However, the origin of the magnetic helicity itself remains to be understood as well (see also below).

In this paper, we propose a new mechanism for a strong and stable magnetic field in magnetars due to a novel instability in the presence of an imbalance between right- and left-handed electrons—the chiral plasma instability. The chiral plasma instability was recently found in the

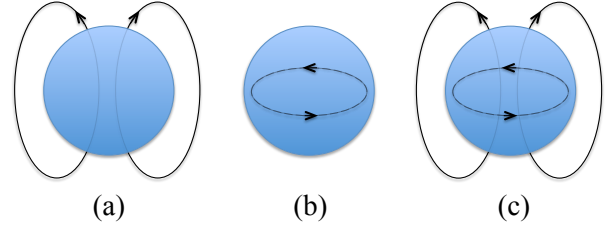


FIG. 1. Configurations of magnetic fields in magnetars: (a) poloidal, (b) toroidal, and (c) linked poloidal-toroidal magnetic fields.

context of electromagnetic and quark-gluon plasmas [8] based on chiral kinetic theory [9]. A related instability had been previously argued for the electroweak theory [10, 11] and for the primordial magnetic field in the early Universe [12]. This instability appears somewhat similar to the Rayleigh-Taylor instability that occurs in the presence of a density imbalance of two fluids at an interface. However, the former is remarkable in that it is a consequence of relativistic and quantum effects related to quantum anomalies [13] unlike the latter.

Our mechanism for a strong and stable magnetic field is based only on the chirality asymmetry of electrons that is inevitably produced in the parity-violating weak process (electron capture) during core collapse of supernovae. The energy of the chirality imbalance is converted to a large magnetic field by the chiral plasma instability. Furthermore, we show that it naturally generates a large magnetic helicity at the same time. To the best of our knowledge, this is the only microscopic mechanism to create a magnetic helicity in magnetars. Note that our mechanism does not require any exotic hadron or quark phases inside compact stars under discussion, such as ferromagnetic nuclear matter [14], ferromagnetic quark matter [15], pion domain walls [16], and so on,

¹ Magnetic helicity is a conserved quantity for a perfect conductor and is conserved approximately with finite conductivity; see the discussion below.

which are essential in some of previous suggestions for the origin of magnetars.

In the following, we neglect the electron mass m_e , as electrons may be regarded as ultrarelativistic at high density, $\mu_e \gg m_e$. We will discuss the possible effects of m_e later on.

Chiral Plasma Instabilities.—Let us briefly review the physical argument of the chiral plasma instability (see Ref. [17] for the detail). For simplicity, we here ignore the effect of dissipation which is not essential to understand the instability itself.

Suppose there is a homogeneous chiral asymmetry between right- and left-handed electrons in the core of compact stars, which we parametrize by a chiral chemical potential $\mu_5 \equiv (\mu_R - \mu_L)/2$. (We shall give an estimate of μ_5 just after the onset of core collapse of supernovae later.) Let us consider a perturbation of a small magnetic field B_z with wavelength λ in a cylindrical coordinate (r, θ, z) (see Fig. 2). In the presence of μ_5 , this magnetic field leads to an electric current in the z direction (called the chiral magnetic effect) [18]:

$$j_z = \frac{2\alpha}{\pi} \mu_5 B_z, \quad (2)$$

where α is the fine structure constant. Intuitively, this current can be understood as follows: to minimize the energy of the system, the spin of an electron is aligned in the same direction as B_z . Remembering the definition of chirality, this means that momentum of a right- (left-)handed fermions is in the same (opposite) direction as B_z , and so is the electric current. Hence, a net electric current flows in the direction of B_z .

Now Ampère’s law states that the current (2) leads to a magnetic field in the θ direction at a distance $R \sim \lambda$ as $B_\theta = \pi \lambda^2 j_z / (2\pi R)$. This in turn induces the current due to the chiral magnetic effect:

$$j_\theta(R) = \frac{2\alpha}{\pi} \mu_5 B_\theta = \left(\frac{2\alpha \mu_5 \lambda}{\pi} \right)^2 \frac{1}{2R} B_z. \quad (3)$$

According to Ampère’s law again, this current gives rise to a magnetic field in the z direction as

$$B'_z = \int dR j_\theta(R) \sim \left(\frac{2\alpha \mu_5 \lambda}{\pi} \right)^2 B_z. \quad (4)$$

So if $\lambda \gtrsim (\alpha \mu_5)^{-1}$, it follows that $B'_z > B_z$: the original magnetic field gives a positive feedback to itself, and it grows exponentially. This is the chiral plasma instability. This unstable mode then reduces μ_5 so that the instability is attenuated [8, 17].

One also finds that this instability generates not only a poloidal magnetic field in the z direction but also a

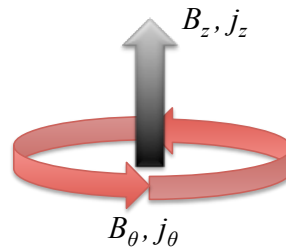


FIG. 2. Physical picture of the chiral plasma instability.

toroidal magnetic field in the θ direction; the resulting configuration has a finite magnetic helicity. Later, we will estimate the magnitude of magnetic helicity from the helicity conservation.

Estimate of the chiral imbalance of electrons.—How large can a magnetic field be due to this mechanism inside compact stars? From now on, we shall provide its estimate based on the neutron density generated during core collapse of supernovae. The core of a compact star is almost “neutronized” at this stage via the parity-violating weak process, and as a result, the largest chiral asymmetry of electrons is created during its evolution [19].

We note that our estimate for the magnetic field below assumes a number of simplifications, so it should be regarded as schematic. Nonetheless, it turns out that the maximal magnetic field due to the chiral plasma instability can be of order 10^{18} G at the core (with a possible small deviation due to the uncertainty of the prefactor), and it might be sufficient enough to explain the large magnetic field $\sim 10^{15}$ G on the surface. Our estimate can, in principle, be made more realistic by including various complications that we will discuss later.

The chirality imbalance of electrons is produced via electron capture inside a core,

$$p + e_L^- \rightarrow n + \nu_L^e, \quad (5)$$

where the subscript L stands for left-handedness. Here only left-handed fermions are involved, as it is described by the V–A type weak interaction. Its inverse process also exists, and reduces the number of neutrons and the chirality imbalance. Note that the other possible processes (thermal neutrino emission) [21] do not change the neutron number nor the chirality imbalance on average.

That huge neutrons are produced in these processes (to form a *neutron* star eventually) means that huge left-handed electrons are “eaten” by protons, leading to the Fermi surface imbalance, $\mu_R > \mu_L$, or a nonzero chiral chemical potential for electrons, $\mu_5 \equiv (\mu_R - \mu_L)/2 > 0$. Because the number of neutrons produced in this process

is equal to the number difference between right- and left-handed electrons, N_5 , we have

$$n_5 \approx \Delta n_n, \quad (6)$$

where n_5 is the chiral density of electrons and Δn_n is the increased neutron density by electron capture. Considering the neutron density inside a neutron star, it is reasonable to take $\Delta n_n \sim (0.1-1) \text{ fm}^{-3}$ at the core. In the natural units $\hbar = c = 1$, $\Delta n_n \sim 0.1-1\Lambda^3$, where we introduced the mass scale $\Lambda = 200 \text{ MeV}$ for later convenience. In the following, we use the natural units unless otherwise stated.

The chiral number density is expressed by the chemical potentials as

$$n_5 = \frac{\mu_5}{3\pi^2}(\mu_5^2 + 3\mu^2) \quad (7)$$

at sufficiently low temperature, where $\mu \equiv (\mu_R + \mu_L)/2$ is the chemical potential associated with U(1) (vector-like) particle number. We here ignored the contribution of the temperature T to the density, because T is at most $\sim 10^{10} \text{ K}$ ($\sim 10^{-2}\Lambda$) at the core, and is negligibly small compared with the contributions of μ and μ_5 (as will be justified below). Recalling that the typical electron chemical potential at the core is $\mu \lesssim \Lambda$, one finds

$$\mu_5 \sim \Lambda. \quad (8)$$

It should be remarked that μ_5 we obtained is the total (or the time-integrated) chiral chemical potential during core collapse. In reality, the production of μ_5 by the process (7) occurs simultaneously with the reduction of μ_5 by the chiral plasma instability (and with the reduction by the electron mass m_e , which we shall argue later).

Estimate of magnetic fields and magnetic helicity.—As explained above, the state with nonzero μ_5 is unstable and decays rapidly by converting it to a magnetic field due to the chiral plasma instability. Assuming this state will decay into a state with $\mu_5 \sim 0$ at saturation, one can estimate the magnitudes of a resulting magnetic field and magnetic helicity from the energy and helicity conservations [8].

The energy conservation requires that the energy density of electrons due to the chiral asymmetry,

$$\Delta E = \frac{1}{4\pi^2}(\mu_5^4 + 6\mu_5^2\mu^2) \quad (9)$$

is equal to that of the magnetic field $(1/2)\Delta B_{\text{inst}}^2$. One can thus estimate the maximal magnetic field generated by this instability as

$$B_{\text{max}} \sim \Lambda^2 \sim 10^{18} \text{ G}, \quad (10)$$

assuming no dissipation of energy and perfect conversion efficiency (see the discussion below). We notice here that the magnitude of the magnetic field is the QCD scale through the relation (6), although the magnetic field itself is produced by the electroweak dynamics. We can translate B_{max} at the core into the magnetic field on the surface from the conservation of a magnetic flux:

$$B_{\text{surface}} \approx \left(\frac{R_{\text{core}}}{R_{\text{star}}}\right)^2 B_{\text{max}}, \quad (11)$$

where R_{core} and R_{star} are the radii of the core and the neutron star itself, respectively; e.g., when $R_{\text{core}}/R_{\text{star}} \sim 10^{-1}$, the maximal surface magnetic field is $B_{\text{surface}} \sim 10^{16} \text{ G}$.

On the other hand, the helicity conservation reads

$$\frac{d}{dt} \left(N_5 + \frac{\alpha}{\pi} \mathcal{H} \right) = 0, \quad N_5 = \int d\mathbf{x} n_5, \quad (12)$$

where N_5 is the global chiral charge of electrons and \mathcal{H} is the magnetic helicity (also called Chern-Simons number in particle physics and mathematics) defined in Eq. (1). In passing, we note that Eq. (12) is the global version of the anomaly relation in quantum electrodynamics [13]:

$$\partial_\mu j^{\mu 5} = \frac{2\alpha}{\pi} \mathbf{E} \cdot \mathbf{B}, \quad (13)$$

where $j^{\mu 5} = \bar{e} \gamma^\mu \gamma^5 e$ is the axial current for electrons. From the helicity conservation, one obtains the magnetic helicity at saturation as

$$\Delta \mathcal{H} = -\frac{\pi}{\alpha} \Delta N_5 \sim -\frac{1}{\alpha} V \Lambda^3 \quad (14)$$

with $V \approx 4\pi R_{\text{core}}^3/3$; so a large magnetic helicity is naturally produced as a consequence of the chiral plasma instability, which then ensures the stability of the strong magnetic field. The detailed configuration of the magnetic field with a large magnetic helicity is under study [22].

We note that any microscopic process concerning the electromagnetic and strong interactions respects parity and cannot generate a parity-odd magnetic helicity; microscopically, a parity-odd quantity can originate from the parity-violating weak interaction alone. However, the weak interaction violates parity in the fermionic sector (leptons), so it cannot generate magnetic helicity directly. It is this chiral plasma instability that converts the parity-odd chiral asymmetry in the fermionic sector to the parity-odd magnetic helicity in the gauge sector.

Note also that the interior of a star could acquire the magnetic helicity *macroscopically* by accident, by losing helicity through the surface in its evolution. However, no such a evidence was observed in magnetohydrodynamics

for a specific initial configuration with $\mathcal{H} = 0$ [5]. At least, our mechanism seems the only to generate magnetic helicity microscopically.

Discussion.—Let us discuss several possible effects we have ignored above, which can modify our simple estimate (10). One potentially important effect is the electron mass m_e which also reduces the chiral asymmetry; for a given μ_5 generated by the process (5), the chirality flipping rate due to m_e is $\Gamma_{\text{mass}} \sim \alpha^2 (m_e/\mu)^2 \mu_5$ [12]. The time scale $\sim \Gamma_{\text{mass}}^{-1}$ is then much larger than that of the chiral plasma instability, $\tau_{\text{inst}} \sim (\alpha^2 \mu_5)^{-1}$ [8], by a factor of $(\mu/m_e)^2 \gg 1$; hence, the effects of m_e is expected to be minor. Another (possibly more) important effect is the conversion efficiency of the chirality imbalance into the magnetic energy which could be less than 100%. For example, the conductivity σ dissipates the energy and makes the magnetic field in Eq. (10) smaller. Also, the nuclear (or QCD) dynamics inside a star may interfere with the electron dynamics and could reduce the magnetic field. Although evaluation of these effects is not easy, it is not entirely unreasonable to expect that the magnetic field induced by the chiral plasma instability can occupy a nonnegligible fraction of the gigantic magnetic field of magnetars.

Finally, we comment on the possible evolution of the large magnetic field after the birth of magnetars by our

mechanism. Remember that the magnetic helicity \mathcal{H} is a strict conserved quantity without dissipation. In reality, the medium has a conductivity σ so that \mathcal{H} is conserved approximately; it is the finiteness of σ that allows magnetic flux tubes to reconnect, which results in the decrease of \mathcal{H} . Therefore, one expects that magnetic fields decay slowly by dissipation and the reconnection which could manifest themselves in the form of giant outbursts.

Conclusion.—In conclusion, we proposed a possible new mechanism for a strong magnetic field with magnetic helicity in magnetars due to the chiral plasma instability. Our mechanism is based only on the chirality imbalance of electrons that is necessarily produced during core collapse. The maximal magnetic field is estimated as $\sim 10^{18}$ G at the core which may be sufficient to explain the magnetic field $\sim 10^{15}$ G on the surface. This large magnetic field is a macroscopic consequence of the relativistic and quantum effects. More realistic calculations, e.g., using magnetohydrodynamics for chiral plasmas, are necessary to reach a definite conclusion. We defer these calculations to future work.

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